The Energy Distribution of the Bianchi Type I Universe

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Abstract

We calculate the energy distribution of an anisotropic model of universe based on the Bianchi type I metric in the Tolman's prescription. The energy due to the matter plus gravitational field is equal to zero. This result agrees with the results of Banerjee and Sen and Xulu. Also, our result supports the viewpoint of Tryon and Rosen.

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1 Introduction

The energy-momentum localization has been a problematic issue since the outset of the theory of relativity. A large number of definitions of the gravitational energy have been given since now. Some of them are coordinate independent and other are coordinate-dependent. An adequate coordinate-independent prescription for energy-momentum localization for all the type of space-times has not given yet in General Relativity. We remark that it is possible to evaluate the energy and momentum distribution by using various energy-momentum complexes. There prevails suspicion that different energy-momentum complexes could give different energy distributions in a given space-time. Virbhadra and his collaborators [1] have considered many space-times and have shown that several energy-momentum complexes give the same and acceptable result for a given space-time. Also, in his recent

paper Virbhadra [2] emphasized that though the energy-momentum complexes are non-tensors under general coordinate transformations, the local conservation laws with them hold in all coordinate systems.

The subject of the energy of the Universe was re-opened by Cooperstock [3] and Rosen [4]. Rosen [4] computed in the Einstein's prescription the energy of a closed homogeneous isotropic universe described by a Friedmann–Robertson–Walker (FRW) metric. The total energy is zero. Johri [5], by using the Landau and Lifshitz energy-momentum complex, found that the total energy of a FRW spatially closed universe is zero at all times irrespective of equations of state of the cosmic fluid. Also, the total energy enclosed within any finite volume of the spatially flat FRW universe is zero at all times. It is known that the Bianchi type I solutions, under a special case, reduce to the spatially flat FRW solutions. The total energy density is found to be zero everywhere. Banerjee and Sen [6] calculated in the Einstein's prescription the total energy density of the Bianchi type I solutions. Also, Xulu [7], by using the Landau and Lifshitz, Papapetrou and Weinberg prescriptions found that the total energy of the Universe in the case of the Bianchi type I model is zero.

The purpose of this paper is to compute the energy of an anisotropic model of universe based on the Bianchi type I metric by using the energy-momentum complex of Tolman. We also make a discussion of the results. We use the geometrized units ($G = 1, c = 1$) and follow the convention that the Latin indices run from 0 to 3.

2 The energy in the Tolman's prescription

We consider the line element [8] which describes a special anisotropic model of universe based on the Bianchi type I metric

$$ds^2 = dt^2 - A^2(t)dx^2 - B^2(t)dy^2 - C^2(t)dz^2, \tag{1}$$

where

$$\begin{aligned} A(t) &= (m_1 s_1 t)^{1/m_1}, \\ B(t) &= (m_2 s_2 t)^{1/m_2}, \\ C(t) &= (m_3 s_3 t)^{1/m_3}. \end{aligned} \tag{2}$$

In (2) m_i and s_i $(i = \overline{1,3})$ are positive constants and we exclude the $m_i = 0$ case.

The metric given by (1) reduces to the spatially flat Friedmann–Robertson–Walker metric in a special case when we have $A(t) = B(t) = C(t) = R(t)$. We define $R(t) = (mst)^{1/m}$ and transforming the line element (1) according to

$$\begin{cases} x = r \sin\theta \cos\varphi, \\ y = r \sin\theta \sin\varphi, \\ z = r \cos\theta. \end{cases} \tag{3}$$

We obtain the line element

$$ds^2 = dt^2 - R^2(t)[dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2)], \tag{4}$$

which describes the spatially flat Friedmann–Robertson–Walker space-time.

The only non-zero components of the energy-momentum tensor (due to the matter) are

$$\begin{aligned} T_1{}^1 &= \left(\frac{m_2 - 1}{m_2^2} + \frac{m_3 - 1}{m_3^2} - \frac{1}{m_2 m_3}\right) t^{-2}, \\ T_2{}^2 &= \left(\frac{m_1 - 1}{m_1^2} + \frac{m_3 - 1}{m_3^2} - \frac{1}{m_1 m_3}\right) t^{-2}, \\ T_3{}^3 &= \left(\frac{m_1 - 1}{m_1^2} + \frac{m_2 - 1}{m_2^2} - \frac{1}{m_1 m_2}\right) t^{-2}, \\ T_0{}^0 &= \left(\frac{m_1 + m_2 + m_3}{m_1 m_2 m_3}\right) t^{-2}. \end{aligned} \tag{5}$$

From (5) it results that the energy density component of the energy-momentum tensor is not zero for the Bianchi type I solutions.

The Tolman's energy-momentum complex [9] is given by

$$\Upsilon_i{}^k = \frac{1}{8\pi} U_i{}^{kl}{}_{,l}\,, \tag{6}$$

where $\Upsilon_0{}^0$ and $\Upsilon_\alpha{}^0$ are the energy and momentum components.

We have

$$U_i{}^{kl} = \sqrt{-g}(-g^{pk} V_{ip}{}^l + \frac{1}{2} g_i^k g^{pm} V_{pm}{}^l), \tag{7}$$

with

$$V_{jk}{}^{i} = -\Gamma^{i}_{jk} + \frac{1}{2} g^{i}_{j} \Gamma^{m}_{mk} + \frac{1}{2} g^{i}_{k} \Gamma^{m}_{mj}. \tag{8}$$

The energy-momentum complex $\Upsilon_i{}^k$ also satisfies the local conservation laws

$$\frac{\partial \Upsilon_i{}^k}{\partial x^k} = 0. \tag{9}$$

The energy and momentum in the Tolman's prescription are given by

$$P_i = \iiint \Upsilon_i{}^0 dx^1 dx^2 dx^3. \tag{10}$$

Using the Gauss's theorem we obtain

$$P_i = \frac{1}{8\pi} \iint U_i^{0\alpha} n_\alpha dS, \tag{11}$$

where $n_\alpha = (x/r, y/r, z/r)$ are the components of a normal vector over an infinitesimal surface element $dS = r^2 \sin\theta d\theta d\varphi$.

By using (6) and (7) we have that all the U_0^{0i} components vanish and we obtain

$$\Upsilon_0^0 = 0. \tag{12}$$

The total energy density (due to the matter plus field) vanishes everywhere.

3 Discussion

The main purpose of the present paper is to show that it is possible to "solve" the problem of the localization of energy in relativity by using the energy-momentum complexes.

The subject of the localization of energy continues to be an open one. Bondi [10] sustained that a nonlocalizable form of energy is not admissible in relativity. A favorable argument for using the energy-momentum complexes to calculate the energy distribution of different models of universe is that they can give the same result for a given space-time. Chang, Nester and Chen [11] showed that the energy-momentum complexes are actually quasilocal and legitimate expressions for the energy-momentum.

As we noted, Rosen [4] found that the total energy of a closed homogeneous isotropic universe described by a FRW metric is zero. Johri [5] showed that the total energy of a FRW spatially closed universe is zero at all times irrespective of equations of state of the cosmic fluid. Also, the total energy enclosed within any finite volume of the spatially flat FRW universe is zero at all times. Banerjee and Sen [6] and Xulu [7] obtained that the total energy density of a model of universe based on the Bianchi type I solutions is zero.

We used a special case of the Bianchi type I metric and obtained a result which agrees with the results of Banerjee and Sen [6] and Xulu [7]. The total energy density vanishes everywhere because the energy contributions from the matter and field inside an arbitrary two-surface, in the case of the anisotropic model based on the Bianchi type I metric, cancel each other. Our result supports the viewpoint of Tryon [12] which assumed that the Universe appeared from nowhere about 10^{10} years ago and the conventional laws of physics need not have been violated at the time of the creation of the Universe. According with his Big Bang model, the Universe is homogeneous, isotropic and closed and consists of matter and anti-matter equally. Also, the net energy of the Universe may be equal to zero. Also, our result supports the calculations of Rosen [4] which agrees with the studies of Tryon. We use the Bianchi I space-time because, as we showed previously, in this case, by using an adequate transformation of coordinates we can reach the line element which describes the spatially flat Friedmann–Robertson–Walker space-time. For an universe described by a metric that can be reduce to the spatially flat FRW metric the total energy is zero [5]-[7].

We completed the investigation of Banerjee and Sen [6] and Xulu [7] with one more energy-momentum complex. The result in this paper sustains the importance of the energy-momentum complexes.

References

[1] J. M. Aguirregabiria, A. Chamorro and K. S. Virbhadra: *Gen. Rel. Grav.* **28** (1996), 1393.

[2] K.S. Virbhadra: *Phys. Rev.* **D60** (1999), 104041.

[3] F. I. Cooperstock: *Gen. Rel. Grav.* **26** (1994), 323.

[4] N. Rosen: *Gen. Rel. Grav.* **26** (1994), 319.

[5] V.B. Johri, D. Kalligos, G.P. Singh and C.W.F. Everitt: *Gen. Rel. Grav.* **27** (1995), 313.

[6] N. Banerjee and S. Sen: *Pramana – J. Phys.* **49** (1997), 609.

[7] S.S. Xulu: *gr-qc*/9910015.

[8] M. S. Berman: *Gen. Rel. Grav.* **8** (1988), 841.

[9] R.C. Tolman: *Relativity, Thermodinamics and Cosmology.* Oxford Univ. Press, London, 1934, 227.

[10] H. Bondi: *Proc. R. Soc., London*, **A427** (1990), 249.

[11] Chia-Chen Chang, J. M. Nester and Chiang-Mei Chen: *Phys. Rev. Lett.* **83** (1999), 1897.

[12] E.P. Tryon: *Nature* **246** (1973), 396.